Estimation of causal effects under the presence of trapdoor variables

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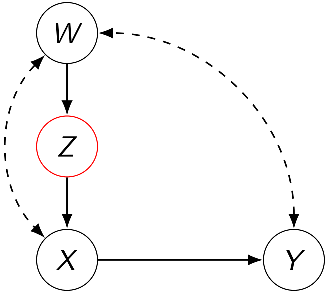
Introduction

- Great progress has been made in **causal identification** research
  - Given a causal graph, produces identifying functional of causal effect (if identifiable)
  - Easy-to-use software implementations available, e.g, the `causaleffect` package in `R` (Tikka and Karvanen 2017)
- Identifiability only indicates the existence of an estimator
  - Does not take into account the potential problems due to finite data and parameter/model uncertainty

Example graph

- We studied (Helske, Tikka, and Karvanen 2021) how to perform **causal estimation** with relatively small data under the presence of *trapdoor variable*
- Y, X, Z, and W observed, dashed arrows are unobserved



$$P(Y \mid \mathrm{do}(X = x)) = \frac{\sum_W P(Y \mid x, Z, W) P(x \mid Z, W) P(W)}{\sum_W P(x \mid Z, W) P(W)}$$

General approach to estimation

- We estimate the full interventional distribution $P(Y \mid \mathrm{do}(X = x))$ by simultaneously estimating the terms of the identifying functional via Bayesian methods.
  - Takes into account the uncertainty in $P(Y \mid \mathrm{do}(X = x))$ due to parameter estimation
  - Avoids the plug-in bias due to the potential nonlinearity of the causal formula.
- However, we still need to figure out what to do with the variable Z...

Trapdoor variable

$$P(Y \mid \mathrm{do}(X = x)) = \frac{\sum_W P(Y \mid x, Z, W)P(x \mid Z, W)P(W)}{\sum_W P(x \mid Z, W)P(W)}.$$

- Tian and Pearl (2002) show that in this graph $P(Y \mid \mathrm{do}(X = x))$ is functionally independent of Z.
- In practice we still must choose some value for Z which can lead to bias.
- Here Z is a *trapdoor variable* with respect to the identifying functional in this graph (for formal definition see Helske, Tikka, and Karvanen (2021)).

Trapdoor variable in linear-Gaussian case

In a linear-Gaussian model, we have

$$W \sim N(a_w, s_w^2),$$
$$(X \mid Z = z, W = w) \sim N(a_x + b_{xz} z + b_{xw} w, s_x^2),$$
$$(Y \mid X = x, Z = z, W = w) \sim N(a_y + b_{yx} x + b_{yz} z + b_{yw} w, s_y^2),$$

where parameters $a_.$, $b_{..}$, and $s_.$ are estimated from the data.

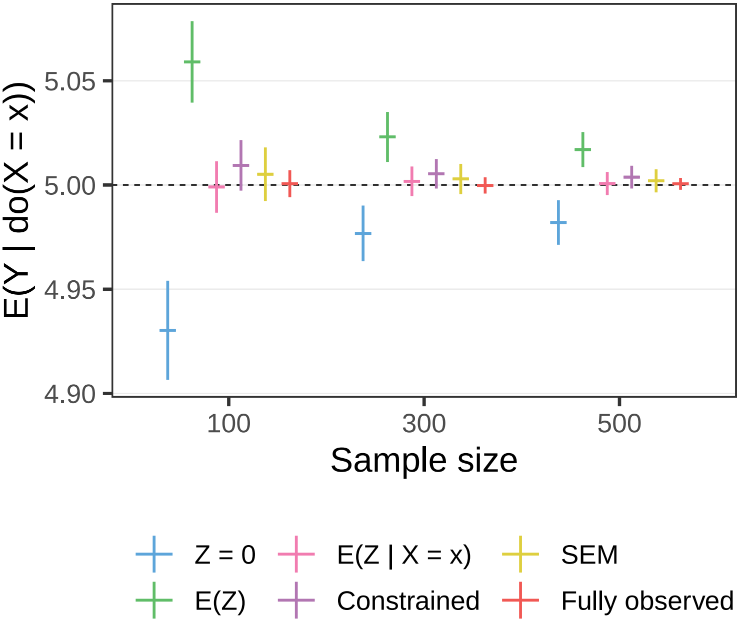
This yields

$$E(Y \mid \mathrm{do}(X = x)) = a_y + \frac{b_{yw} s_x^2}{b_{xw}^2 s_w^2 + s_x^2} a_w - \frac{b_{yw} b_{xw} s_w^2}{b_{xw}^2 s_w^2 + s_x^2} a_x$$
$$+ \left(b_{yx} + \frac{b_{yw} b_{xw} s_w^2}{b_{xw}^2 s_w^2 + s_x^2} \right) x$$
$$+ \left(b_{yz} - \frac{b_{yw} b_{xw} s_w^2}{b_{xw}^2 s_w^2 + s_x^2} b_{xz} \right) z$$

Simulation experiment

1. Fix the trapdoor variable Z to the constant $z = 0$.
2. Also estimate $P(Z)$ and use $z = E(Z)$.
3. As above, but estimate $P(Z \mid X = x)$ and use $z = E(Z \mid X = x)$.
4. Use a constraint $b_{yz} = (b_{xz} b_{yw} b_{xw} s_w^2)/(b_{xw}^2 s_w^2 + s_x^2)$ so that the contribution of z is fixed to zero (red term in previous slide is zero).
5. Use a linear-Gaussian structural equation model (SEM).

Results (1000 replications from data generating process)



References

Helske, Jouni, Santtu Tikka, and Juha Karvanen. 2021. "Estimation of Causal Effects with Small Data Under the Presence of Trapdoor Variables." *To Appear in Journal of Royal Statistical Society, Series A*. https://arxiv.org/abs/2003.03187.

Tian, Jin, and Judea Pearl. 2002. "On the Testable Implications of Causal Models with Hidden Variables." In *Proceedings of the 18th Conference on Uncertainty in Artificial Intelligence*, 519–27. Morgan Kaufmann.

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